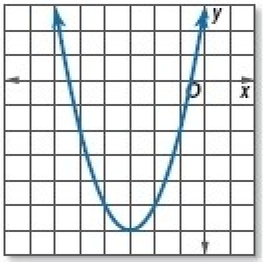
9-1 Graphing Quadratic Functions

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.

28. 

SOLUTION:

Find the vertex.

Because the parabola opens up, the vertex is located at the minimum point of the parabola. It is located at (−3, −6).

Find the axis of symmetry.

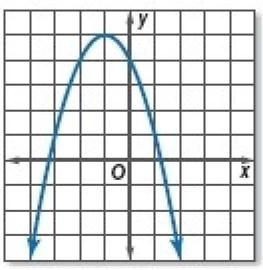
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at x = –3.

Find the y-intercept.

The y-intercept is the point where the graph intersects the y-axis. It is located at (0, 3), so the y-intercept is 3.

ANSWER:

vertex (−3, −6), axis of symmetry $x = -3$, y-intercept 3

30. 

SOLUTION:

Find the vertex.

Because the parabola opens down, the vertex is located at the maximum point of the parabola. It is located at (−1, 5).

Find the axis of symmetry.

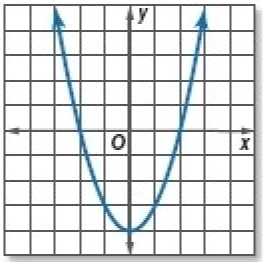
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at x = –1.

Find the y-intercept.

The y-intercept is the point where the graph intersects the y-axis. It is located at (0, 4), so the y-intercept is 4.

ANSWER:

vertex (−1, 5), axis of symmetry $x = -1$, y-intercept 4

32. 

SOLUTION:

Find the vertex.

Because the parabola opens up, the vertex is located at the minimum point of the parabola. It is located at (0, −4).

Find the axis of symmetry.

The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at $x = 0$.

Find the y-intercept.

The y-intercept is the point where the graph intersects the y-axis. It is located at (0, –4), so the y-intercept is –4.

ANSWER:

vertex (0, −4), axis of symmetry $x = 0$, y-intercept −4

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

34. $y = x^2 + 8x + 10$

SOLUTION:

Find the vertex.

In the equation $y = x^2 + 8x + 10$, $a = 1$, $b = 8$, and $c = 10$.

The x-coordinate of the vertex is $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a}$$
$$x = \frac{-8}{2 \bullet (1)}$$
$$x = \frac{-8}{2}$$
$$x = -4$$

The x-coordinate of the vertex is $x = –4$. Substitute the x-coordinate of the vertex into the original equation to find the value of y.

$$f(x) = x^2 + 8x + 10$$
$$f(-4) = (-4)^2 + 8(-4) + 10$$
$$f(-4) = 16 - 32 + 10$$
$$f(-4) = -6$$

The vertex is at (–4, –6).

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at $x = –4$.

Find the y-intercept.

The y-intercept always occurs at (0, c). Since $c = 10$ for this equation, the y-intercept is located at (0, 10).

ANSWER:

vertex (−4, −6), axis of symmetry $x = −4$, y-intercept 10

38. $y = 5x^2 + 20x + 10$

SOLUTION:

Find the vertex.

In the equation $y = 5x^2 + 20x + 10$, $a = 5$, $b = 20$, and $c = 10$.

The x-coordinate of the vertex is $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a}$$
$$x = \frac{-(20)}{2 \bullet (5)}$$
$$x = \frac{-20}{10}$$
$$x = -2$$

The x-coordinate of the vertex is $x = -2$. Substitute the x-coordinate of the vertex into the original equation to find the value of y.

$$f(x) = 5x^2 + 20x + 10$$
$$f(-2) = 5(-2)^2 + 20(-2) + 10$$
$$f(-2) = 20 - 40 + 10$$
$$f(-2) = -10$$

The vertex is at (–2, –10).

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at $x = -2$.

Find the y-intercept.

The y-intercept always occurs at (0, c). Since $c = 10$ for this equation, the y-intercept is located at (0, 10).

ANSWER:

vertex (−2, −10), axis of symmetry $x = -2$, y-intercept 10

Consider each function.
a. Determine whether the function has a *maximum* or *minimum* value.
b. State the maximum or minimum value.
c. What are the domain and range of the function?

43. $y = -2x^2 - 8x + 1$

SOLUTION:

a. For $y = -2x^2 - 8x + 1$, $a = -2$, $b = -8$, and $c = 1$. Because a is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a}$$
$$x = \frac{-(-8)}{2 \cdot (-2)}$$
$$x = \frac{8}{-4}$$
$$x = -2$$

The x-coordinate of the vertex is $x = -2$. Substitute this value into the function to find the y-coordinate.

$$f(x) = -2x^2 - 8x + 1$$
$$f(-2) = -2(-2)^2 - 8(-2) + 1$$
$$f(-2) = -8 + 16 + 1$$
$$f(-2) = 9$$

The maximum value is 9.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or $\{f(x) | f(x) \leq 9\}$.

ANSWER:

a. maximum

b. 9

c. D = {all real numbers},
R = $\{f(x) | f(x) \leq 9\}$

44. $y = x^2 + 4x - 5$

SOLUTION:

a. For $y = x^2 + 4x - 5$, $a = 1$, $b = 4$, and $c = –5$. Because a is positive, the graph opens upward, so the function has a minimum value.

b. The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a}$$
$$x = \frac{-(4)}{2 \cdot (1)}$$
$$x = \frac{-4}{2}$$
$$x = -2$$

The x-coordinate of the vertex is $x = –2$. Substitute this value into the function to find the y-coordinate.

$$f(x) = x^2 + 4x - 5$$
$$f(-2) = (-2)^2 + 4(-2) - 5$$
$$f(-2) = 4 - 8 - 5$$
$$f(-2) = -9$$

The minimum value is –9.

c. The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or $\{f(x) | f(x) \geq -9\}$.

ANSWER:

a. minimum
b. −9
c. D = {all real numbers},
R = $\{f(x) | f(x) \geq -9\}$

Graph each function.

52. $y = -3x^2 + 6x - 4$

SOLUTION:

Step 1 Find the equation of the axis of symmetry. For $y = -3x^2 + 6x - 4$, $a = –3$, $b = 6$, and $c = –4$.

$$x = \frac{-b}{2a}$$
$$x = \frac{-(6)}{2 \cdot (-3)}$$
$$x = \frac{-6}{-6}$$
$$x = 1$$

Step 2 Find the vertex, and determine whether it is a maximum or minimum.
The x-coordinate of the vertex is $x = 1$. Substitute the x-coordinate of the vertex into the original equation to find the value of y.

$f(x) = -3x^2 + 6x - 4$

$f(1) = -3(1)^2 + 6(1) - 4$

$f(1) = -3 + 6 - 4$

$f(1) = -1$

The vertex lies at (1, –1). Because *a* is negative, the graph opens down, and the vertex is a maximum.

Step 3 Find the *y*-intercept.
Use the original equation, and substitute 0 for *x*.

$y = -3x^2 + 6x - 4$

$y = -3(0)^2 + 6(0) - 4$

$y = 0 + 0 - 4$

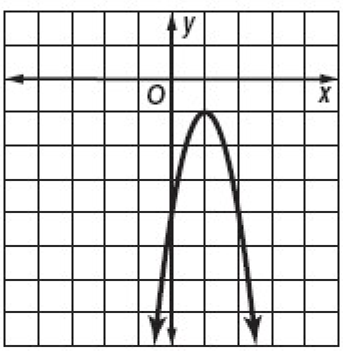
$y = -4$

The *y*-intercept is (0, –4).

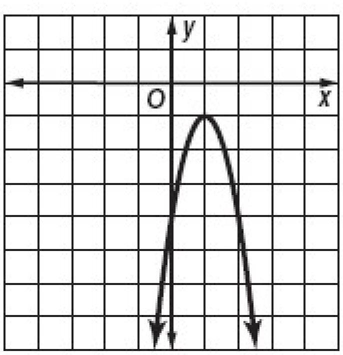
Step 4 The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same *y*-value.

x	–1	0	1	2	3
y	–13	–4	–1	–4	–13

Step 5 Connect the points with a smooth curve.



ANSWER:



63. **GOLF** The average amateur golfer can hit the ball with an initial upward velocity of 31.3 meters per second. The height can be modeled by the equation $h = -4.9t^2 + 31.3t$, where *h* is the height of the ball, in feet, after *t* seconds.

a. Graph this equation.

b. At what height is the ball hit?

c. What is the maximum height of the ball?
d. How long did it take for the ball to hit the ground?
e. State a reasonable range and domain for this situation.

SOLUTION:

a. **Step 1** Find the equation of the axis of symmetry. $h = -4.9t^2 + 31.3t$, $a = -4.9$ and $b = 31.3$.

$$t = \frac{-b}{2a}$$
$$t = \frac{-(31.3)}{2 \cdot (-4.9)}$$
$$t = \frac{-31.3}{-9.8}$$
$$t \approx 3.2$$

Step 2 Find the vertex, and determine whether it is a maximum or minimum.
The *t*-coordinate of the vertex is $t = 3.2$. Substitute the *t*-coordinate of the vertex into the original equation to find the value of *h*.

$$h = -4.9t^2 + 31.3t$$
$$h = -4.9(3.2)^2 + 31.3(3.2)$$
$$h = -50.2 + 100.2$$
$$h \approx 50$$

The vertex lies at about (3.2, 50). Because *a* is negative, the graph opens down, and the vertex is a maximum.

Step 3 Find the *h*-intercept.
Use the original equation, and substitute 0 for *t*.

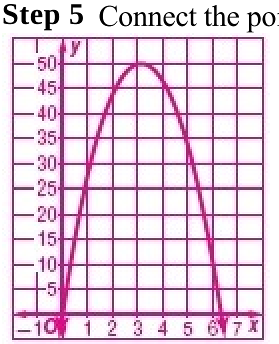
$$h = -4.9t^2 + 31.3t$$
$$h = -4.9(0)^2 + 31.3(0)$$
$$h = 0 + 0$$
$$h = 0$$

The *h*-intercept is (0, 0).

Step 4 The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same *h*-value.

t	0	3.2	6.4
h	0	50	0

Step 5 Connect the points with a smooth curve.



b. The ball is hit when the time is zero, or at the *t*-intercept. Since the *t*-intercept is (0, 0), the ball is hit at 0 meters.

c. The maximum height of the ball is at the vertex. The vertex is (3.2, 50), so the maximum height of the ball is 50 meters.
d. It takes the ball about 3.2 seconds to reach the vertex, and another 3.2 seconds to come down. Therefore, it takes about 3.2 + 3.2 or about 6.4 seconds to reach the ground.
e. Since the time is zero when the ball is hit and 6.4 when it reaches the ground, the domain is D = $\{t|0 \le t \le 6.4\}$. The ball starts at 0 meters and reaches a maximum height of about 50 meters, so the R = $\{h|0 \le h \le 50.0\}$

ANSWER:

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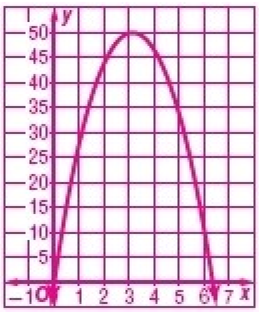

b. 0 m
c. ≈50.0 m
d. ≈6.4 s
e. D = $\{t|0 \le t \le 6.4\}$; R = $\{h|0 \le h \le 50.0\}$

64. **FUNDRAISING** The marching band is selling poinsettias to buy new uniforms. Last year the band charged $5 each, and they sold 150. They want to increase the price this year, and they expect to lose 10 sales for each $1 increase. The sales revenue *R*, in dollars, generated by selling the poinsettias is predicted by the function $R = (5 + p)(150 - 10p)$, where p is the number of $1 price increases.

a. Write the function in standard form.

b. Find the maximum value of the function.

c. At what price should the poinsettias be sold to generate the most sales revenue? Explain your reasoning.

SOLUTION:

a.

$$R = (5 + p)(150 - 10p)$$
$$R = (5)(150) - (5)(10p) + (p)(150) - (p)(10p)$$
$$R = 750 - 50p + 150p - 10p^2$$
$$R = -10p^2 + 100p + 750$$

b. The maximum value of the function occurs at the vertex. In the function $R = -10p^2 + 100p + 750$, $a = -10$, $b = 100$, and $c = 750$.

The *x*-coordinate of the vertex is $p = \frac{-b}{2a}$.

$$p = \frac{-b}{2a}$$
$$p = \frac{-100}{2 \bullet (-10)}$$
$$p = \frac{-100}{-20}$$
$$p = 5$$

The p-coordinate of the vertex is $p = 5$. Substitute the p-coordinate of the vertex into the original equation to find the value of *R*.

$$R = -10p^2 + 100p + 750$$
$$R = -10(5)^2 + 100(5) + 750$$
$$R = -250 + 500 + 750$$
$$R = 1000$$

The maximum value of the function is 1000.

c. The maximum revenue of $1000, is generated when $p = 5$, so by five $1 increases. Since the original price was $5, the new price should be 5 + 5 or $10.

ANSWER:

a. $R = -10p^2 + 100p + 750$

b. 1000

c. $10; Sample answer: The maximum revenue is generated by five $1 increases. The original price was $5, so the new price should be $10.