Appendix A

Important graphs to memorize and graph transformations

$y = x^2$

$y = x^3$

$y = |x|$

$y = \sqrt{x}$

$y = \dfrac{1}{x}$

$y = \ln x$

$y = e^x$

$y = \cos x$

$y = \sin x$

$$y = \tan x$$

$$y = \cot x$$
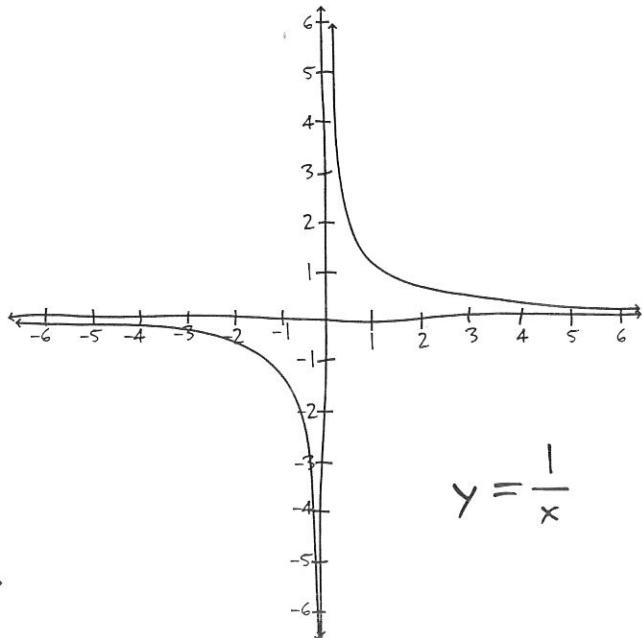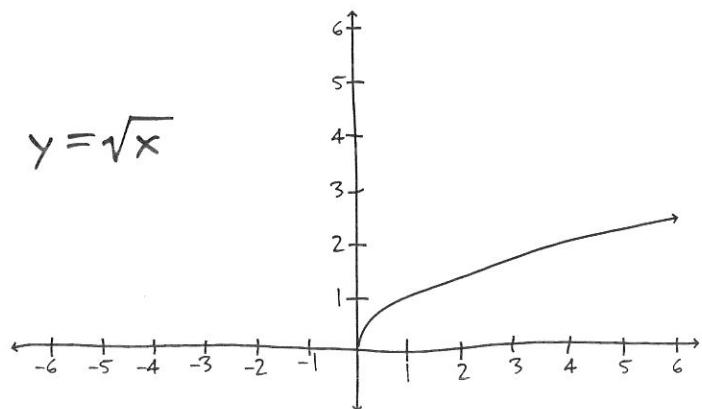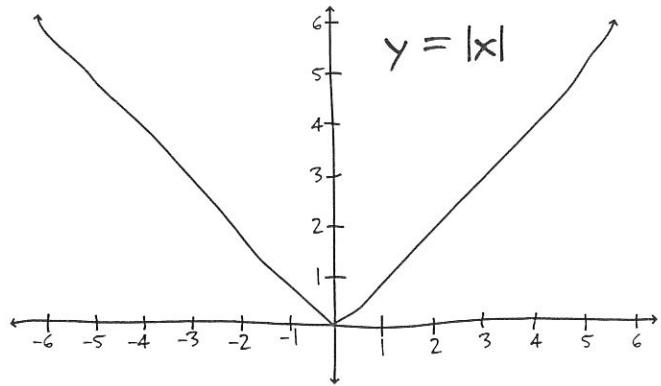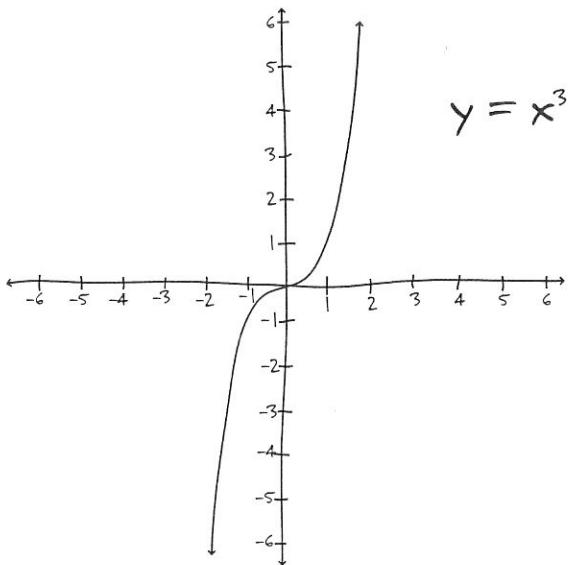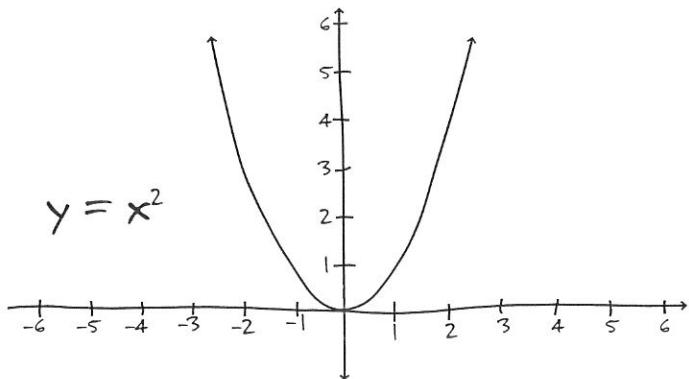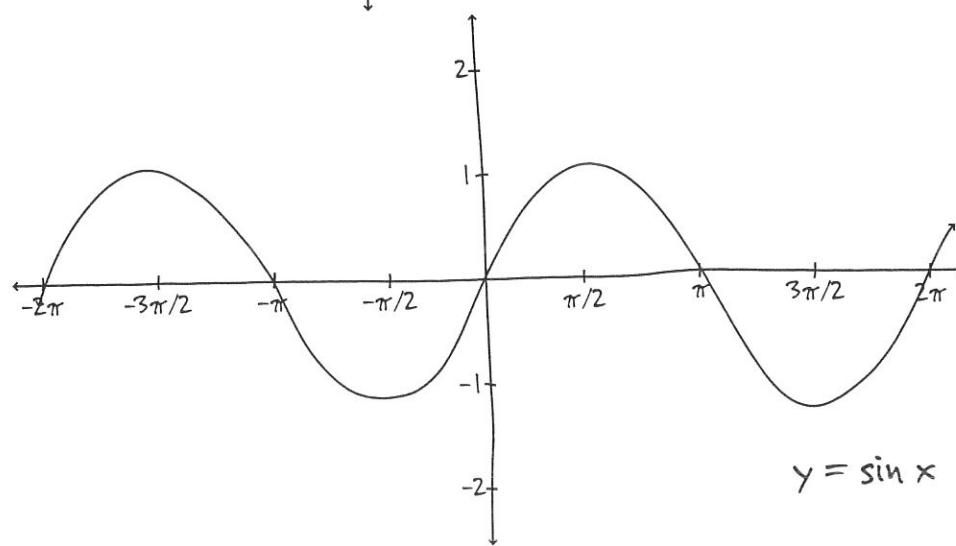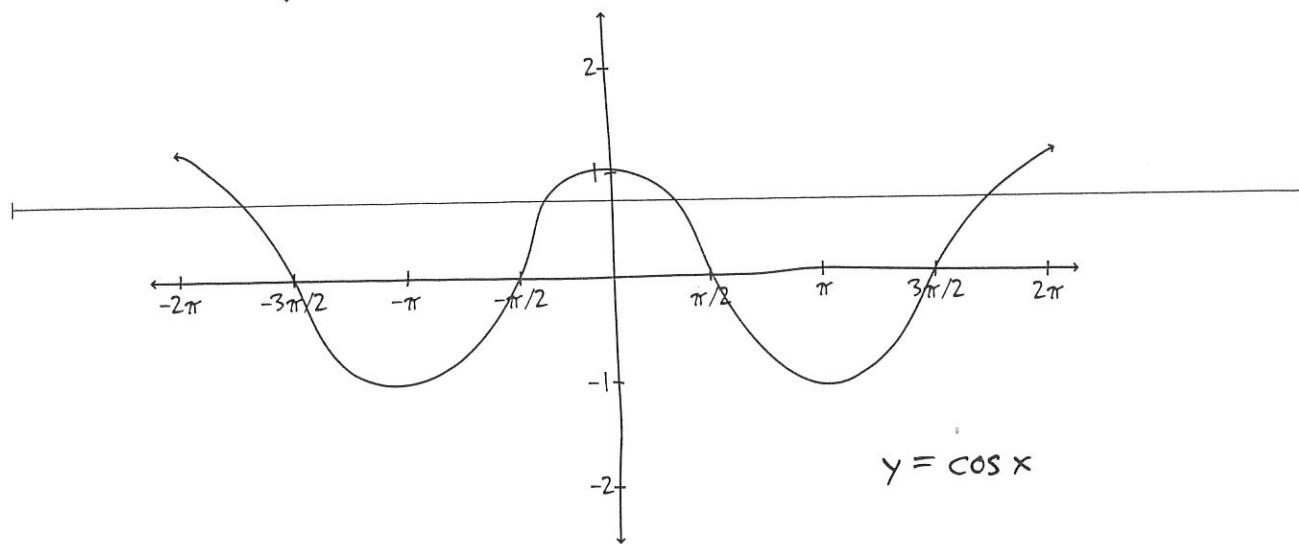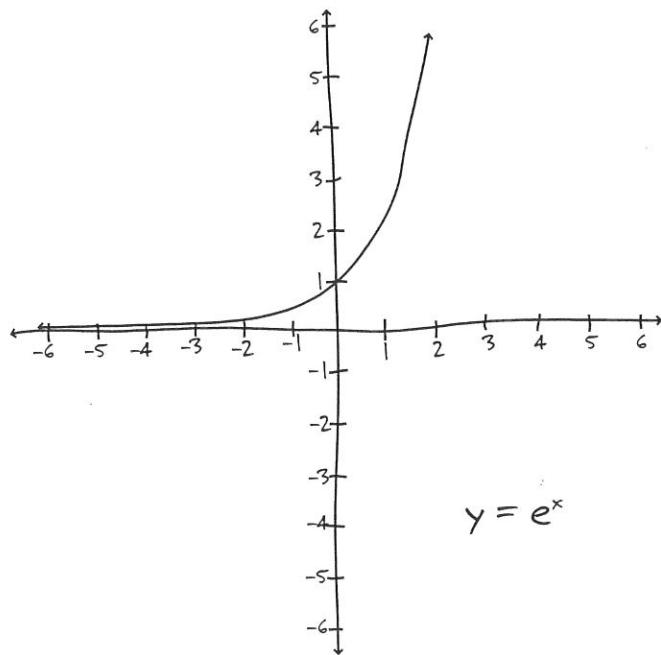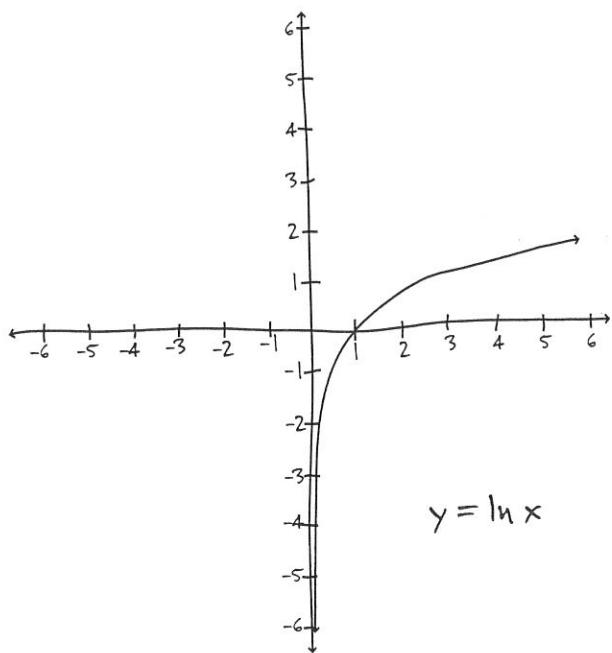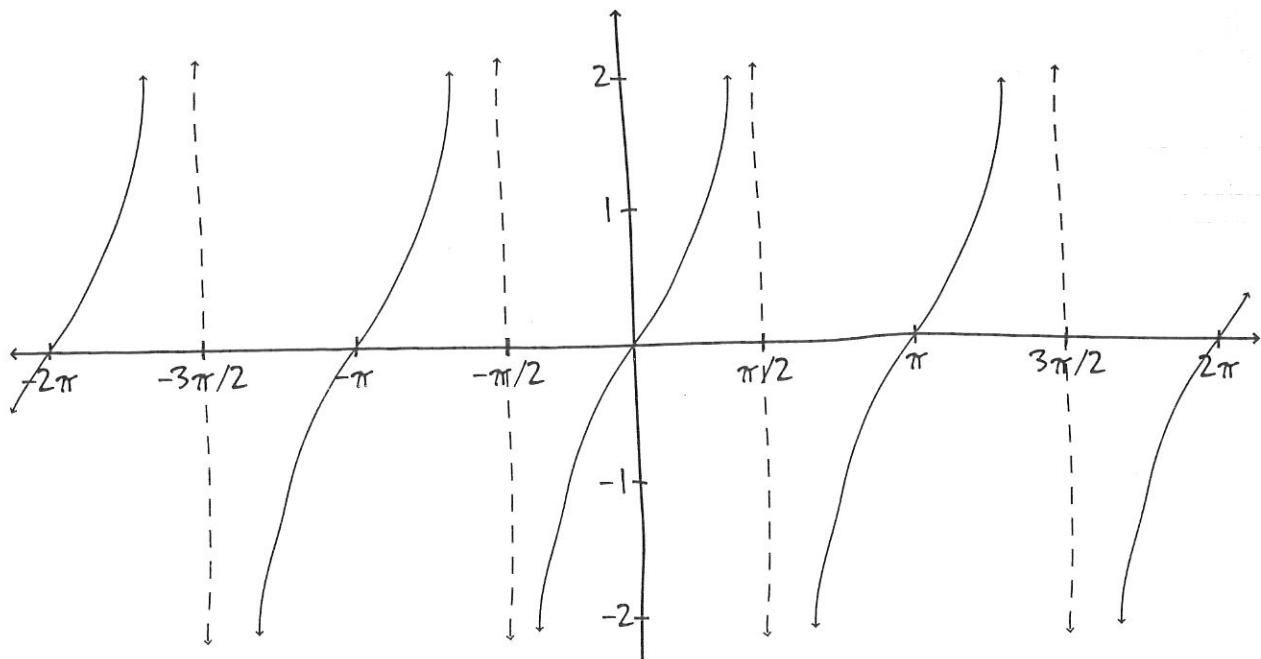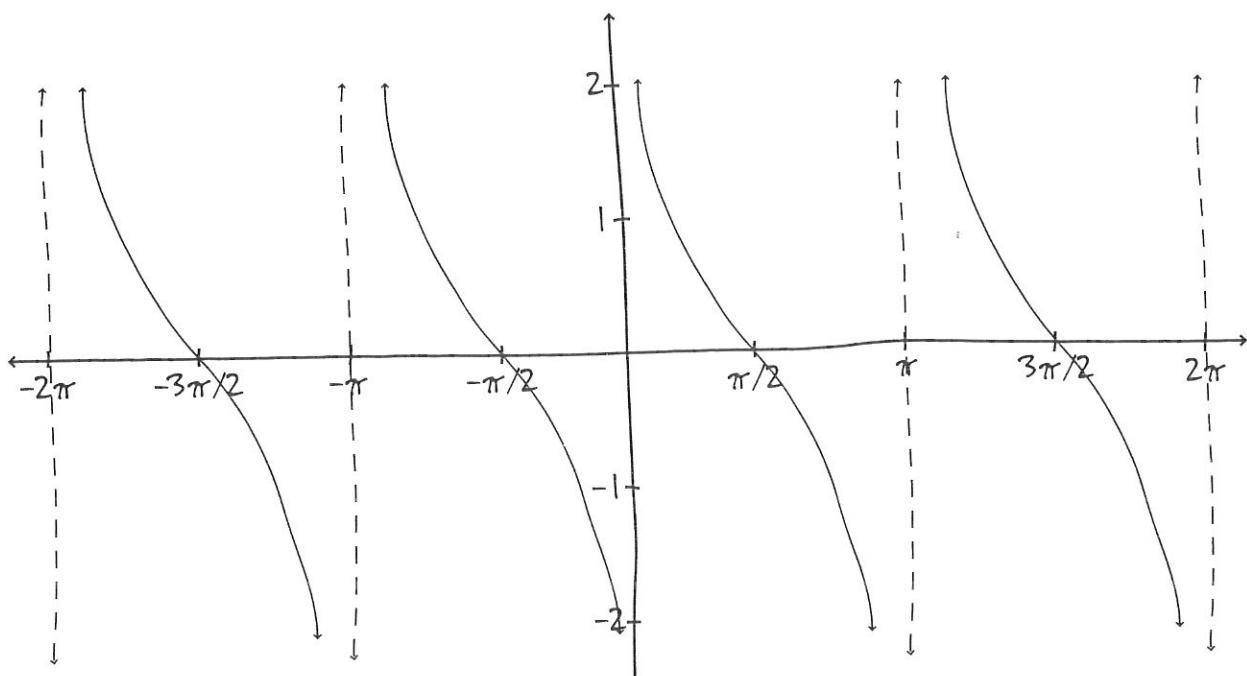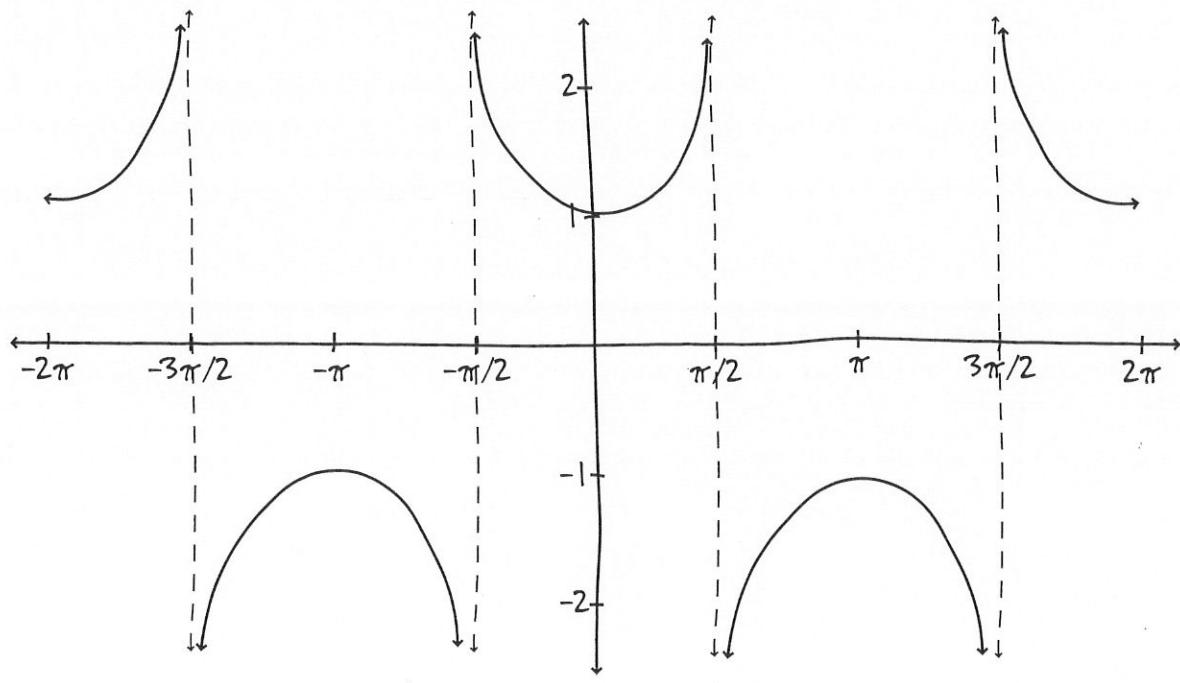
$$y = \sec x$$
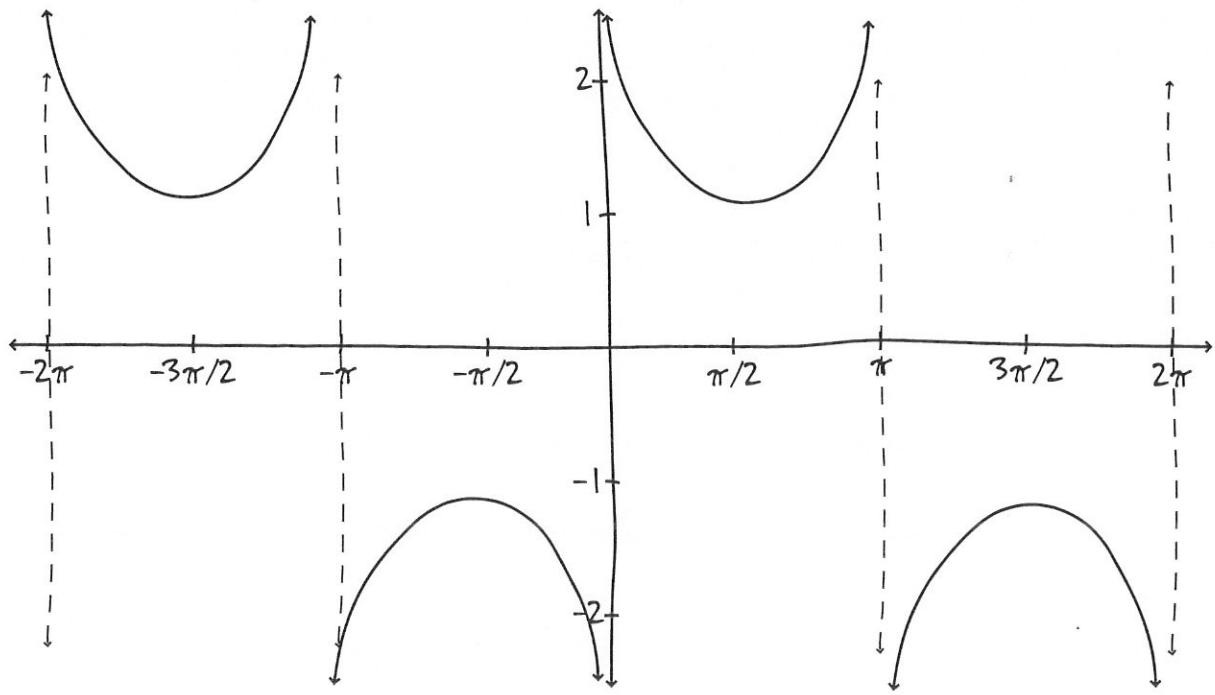
$$y = \csc x$$

How constants transform a function graph

Absolute value of $a \cdot f(bx + c) + d$ below the x-axis across the x-axis reflects portions of $f(x)$

b > 1: squishes $f(x)$ horizontally
0 < b < 1: stretches $f(x)$ horizontally
−1 < b < 0: flips $f(x)$ over y-axis and stretches it horizontally
b < −1: flips $f(x)$ over y-axis and squishes it horizontally

c > 0: shifts graph of $f(x)$ to the left
c < 0: shifts graph of $f(x)$ to the right

$$y = a \cdot f(bx + c) + d$$

a > 1: stretches $f(x)$ vertically
0 < a < 1: squishes $f(x)$ vertically
−1 < a < 0: flips $f(x)$ over x-axis and squishes it vertically
a < −1: flips $f(x)$ over x-axis and stretches it vertically

d > 0: shifts graph of $f(x)$ up
d < 0: shifts graph of $f(x)$ down

Absolute value of $(bx + c)$ erases graph left of y-axis and replaces it with reflection of $f(x)$ across the y-axis

Appendix B

The unit circle

$$90° = \frac{\pi}{2} = (0,1)$$

$$120° = \frac{2\pi}{3} = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$135° = \frac{3\pi}{4} = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$150° = \frac{5\pi}{6} = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$60° = \frac{\pi}{3} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$45° = \frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$30° = \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$180° = \pi = (-1,0)$$

$$0° = 360° = 0 = 2\pi = (1,0)$$

$$210° = \frac{7\pi}{6} = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$225° = \frac{5\pi}{4} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$240° = \frac{4\pi}{3} = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$270° = \frac{3\pi}{2} = (0,-1)$$

$$300° = \frac{5\pi}{3} = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$315° = \frac{7\pi}{4} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$330° = \frac{11\pi}{6} = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$
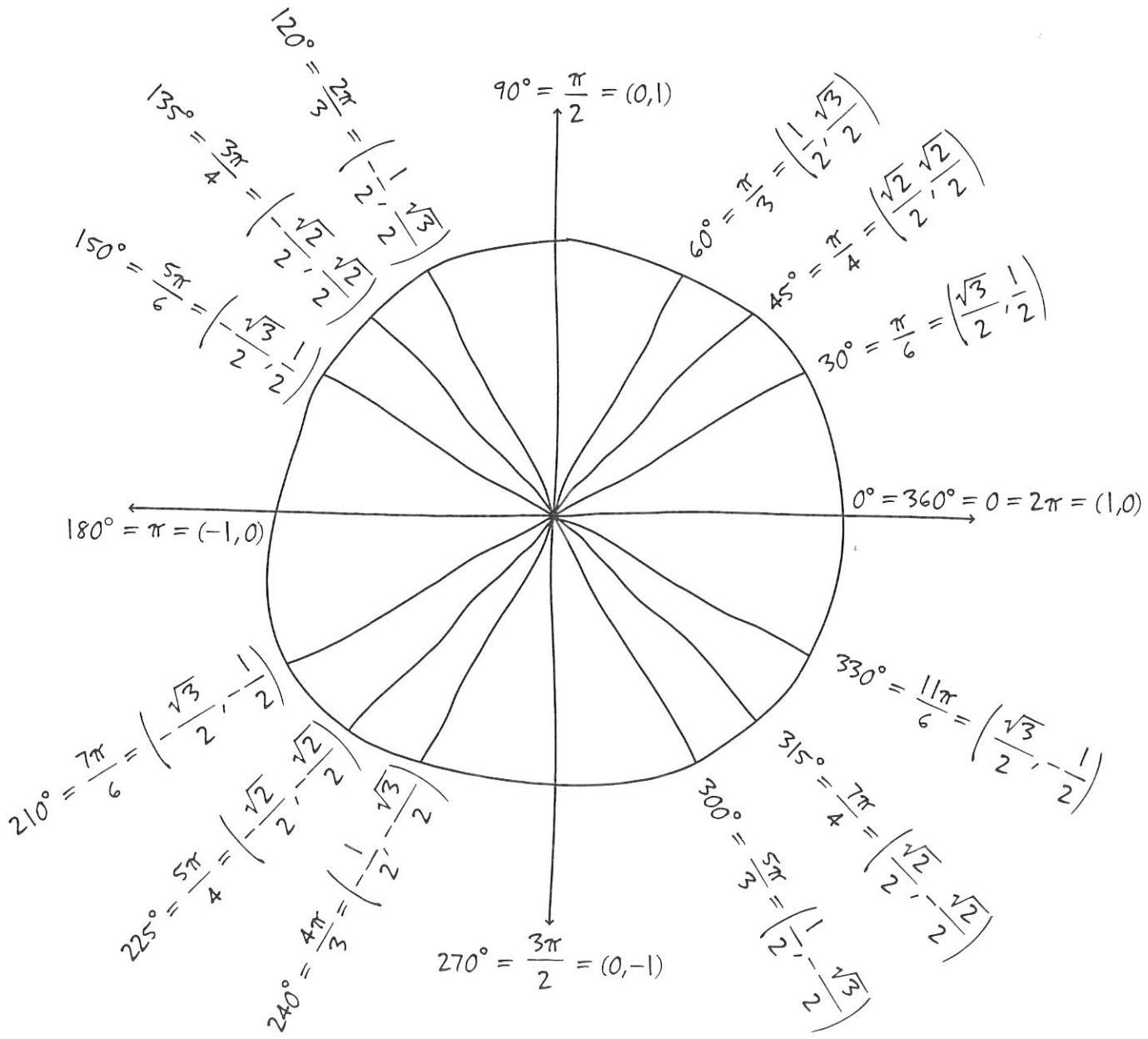
Appendix C
Trigonometric identities

Reciprocal Identities

$$\cos x = \frac{1}{\sec x}$$

$$\sin x = \frac{1}{\csc x}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

Cofunction Identities

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Pythagorean Identities

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double Angle Identities

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Sum and Difference Identities

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Product-to-Sum Identities

$$\cos A \cos B = \frac{\cos(A - B) + \cos(A + B)}{2}$$

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

$$\sin A \cos B = \frac{\sin(A + B) + \sin(A - B)}{2}$$

Power-Reduction Formulas

$$\cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

Appendix D
Derivative Formulas

Trig Derivatives

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Exponential/Log Derivatives

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = (\ln a)\, a^x$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x(\ln a)}$$

Inverse Trig Derivatives

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\text{arccot}\, x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\text{arcsec}\, x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\text{arccsc}\, x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

Appendix E
Antiderivative Formulas

Trig Antiderivatives

$\int \cos x \, dx = \sin x + C$

$\int \sin x \, dx = -\cos x + C$

$\int \tan x \, dx = -\ln |\cos x| + C$

$\int \cot x \, dx = \ln |\sin x| + C$

$\int \sec x \, dx = \ln |\sec x + \tan x| + C$

$\int \csc x \, dx = -\ln |\csc x + \cot x| + C$

Exponential/Log Antiderivatives

$\int e^x \, dx = e^x + C$

$\int \ln x \, dx = x \ln x - x + C$

Inverse Trig Antiderivatives

$\int \dfrac{du}{\sqrt{a^2 - u^2}} = \arcsin \dfrac{u}{a} + C$

$\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a} \arctan \dfrac{u}{a} + C$

$\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a} \text{arcsec} \dfrac{|u|}{a} + C$