CONTINUITY → CAN YOU WRITE w/o LIFTING PENCIL.... (DRAW)

↳ MUCH LIKE DOMAIN, THINK ABOUT WHERE ITS NOT CONTINUOUS....

⇒ WHERE ARE THE "ISSUES"?

→ SQRT, FRACTION DENOM?, GRAPH....

TYPES OF DISCONTINUITY:

(1) REMOVABLE DISCONT. : • GRAPHICALLY, ITS A HOLE

• ALGEBRAICALLY, ITS A DENOMINATOR THAT CAN BE CANCELLED OUT

* BY ALGEBRAICALLY CANCELLING OUT A DENOMINATOR THAT HAD YIELDED A DISCONTINUITY DUE TO DIVISION BY ZERO, YOU ELIMINATE THE ISSUE... THEREFORE, YOU PLUG THE HOLE BACK IN. **

(2) NON-REMOVABLE (ESSENTIAL) DISCONT: • GRAPHICALLY: — JUMP → STEP FUNCTION (GREATEST INTEGER FNC)

— SQRTS → GRAPH ENDS

– ALGEBRAICALLY, YOU CANNOT CANCEL THE DENOMINATOR...

** IF DENOMINATOR CANNOT BE CANCELLED, YOU HAVE A VERTICAL ASYMPTOTE !!! **

EX: $y = \sqrt{9 - x^2}$ → ORIGINALLY... $x^2 + y^2 = 9$

CIRCLE, $r = 3$

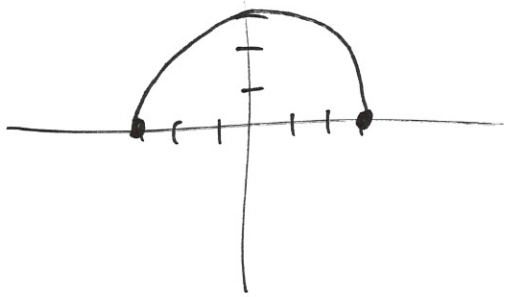
CENTER $(0,0)$

NOT A FUNCTION!

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$

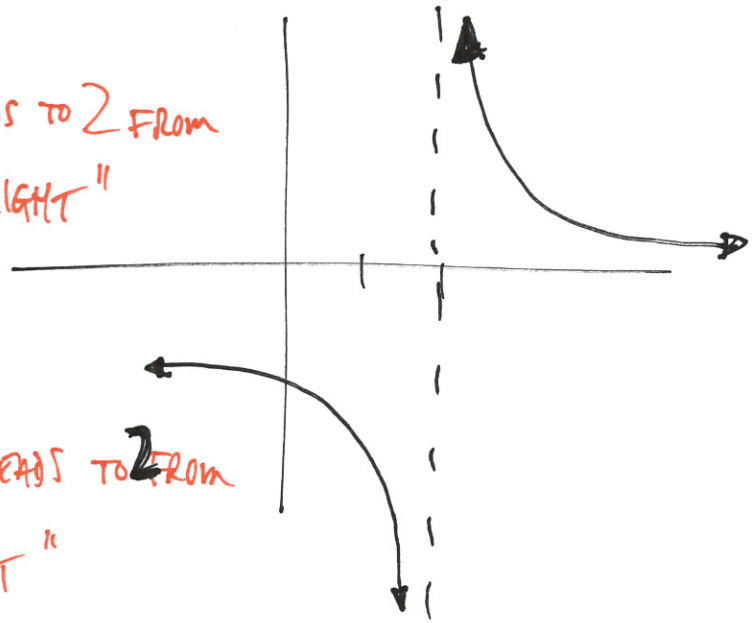
→ IS ONLY THE UPPER 'HALF'



ONE-SIDED LIMITS

$\lim_{x \to 2^+} f(x)$ "limit as x HEADS TO 2 FROM RIGHT"

$\lim_{x \to 2^-} f(x)$ "limit as x HEADS TO 2 FROM LEFT"

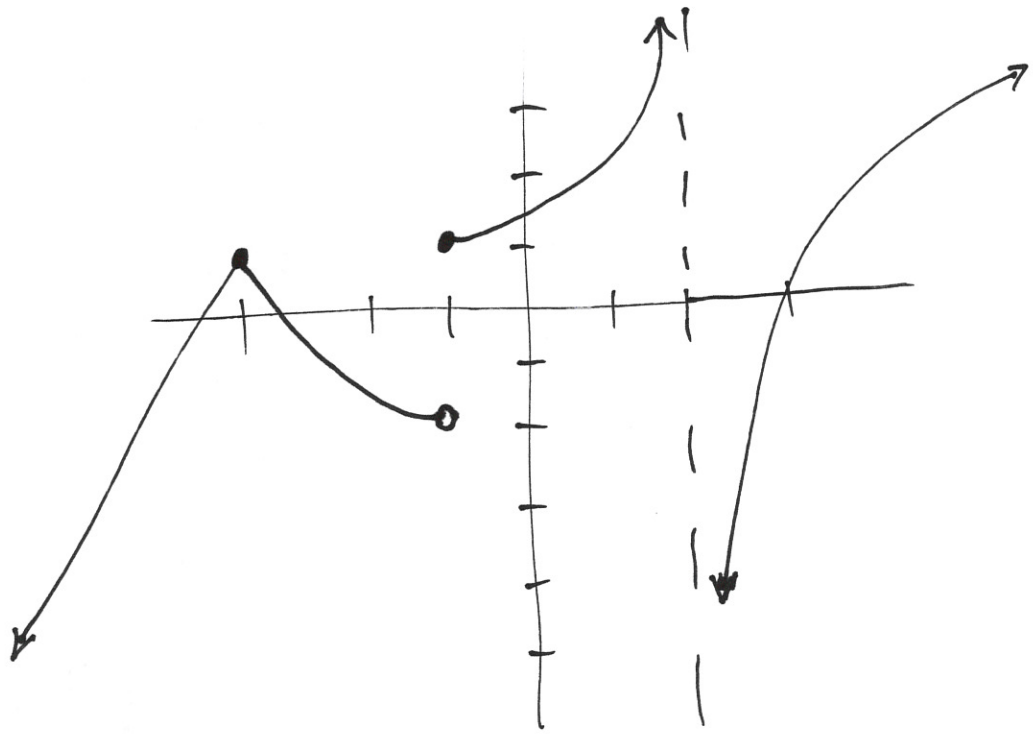


$$\lim_{x \to 2^+} f(x) = +\infty$$

$$\lim_{x \to 2^-} f(x) = -\infty$$

$$\lim_{x \to 2} f(x) = \text{DNE} \quad \text{(OVERALL LIMIT)}$$

EX:



a. $\lim_{x \to -3^+} f(x) = 1$

b. $\lim_{x \to -3^-} f(x) = 1$

c. $\lim_{x \to -3} f(x) = 1$

d. $f(-3) = 1$ $(-3, 1)$

e. $\lim_{x \to -1^-} f(x) = -2$

f. $\lim_{x \to -1^+} f(x) = 1$

g. $\lim_{x \to -1} f(x) = \text{DNE}$ $(e \neq f)$

h. $f(-1) = 1$

i. $\lim_{x \to 2^-} f(x) = +\infty$

j. $\lim_{x \to 2^+} f(x) = -\infty$

k. $\lim_{x \to 2} f(x) = \text{DNE}$ $(i \neq j)$

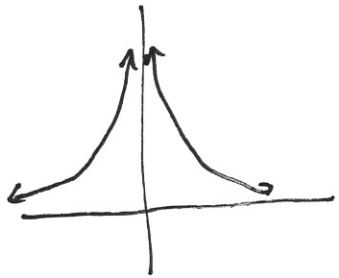
l. $f(2) =$ UNDEFINED (value of V.A.)

THE LIE OF INFINITE LIMITS: $\lim_{x \to c} f(x) = \infty$

- "EQUALS INFINITY" .... NOTHING CAN EQUAL INFINITY... WE JUST DON'T HAVE A BETTER WAY TO WRITE IT
- IT DOES NOT MEAN THE LIMIT EXISTS !!!
- TECHNICALLY, ITS TELLING YOU IT FAILS TO EXIST BASED ON ITS UNBOUNDED BEHAVIOR... LIKE A WILD STALLION!

THIS IS FUN AND EASY:

EX: $\lim_{x \to 0} \frac{1}{x^2}$ CANNOT BE D.S. AND NO WAY TO SIMPLIFY SO CHECK THE GRAPH.... (BEWARE OF V.A. ON TI-83)



$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$

$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$

$\therefore \lim_{x \to 0} \frac{1}{x^2} = \infty$

$\frac{2}{\infty} = 0$

POINTS TO NOTE:

EXAMPLES:

$\infty + 1 = \infty$

$\infty - 1 = \infty$

$2(\infty) = \infty$

$\frac{\infty}{2} = \infty$

$\infty + \infty = 2\infty = \infty$

ALSO: $\frac{0}{0}$

INDETERMINANT FORMS

BUT:

$\infty \cdot 0 \neq 0$ = IND

$\infty^0 \neq 1$ IND

$\frac{\infty}{\infty} \neq 1$ IND

$1^\infty \neq 1$ IND

$\infty - \infty \neq 0$ IND

END BEHAVIOR AND LIMITS AT INFINITY

1 up 1 down, 2 up, 2 down:

1. IF DEGREE ON TOP < DEGREE ON BOTTOM, LIMIT OF RATIONAL (FRACTION) IS 0.... X-AXIS IS HORIZONTAL ASYMPTOTE.

EX: $\lim_{x\to\infty} \frac{2x+5}{3x^2+1} = \frac{\infty}{\infty}$ (NO!!)

AS YOU HEAD TO THE RIGHT END.... ASK ME IF YOU DON'T GET IT.

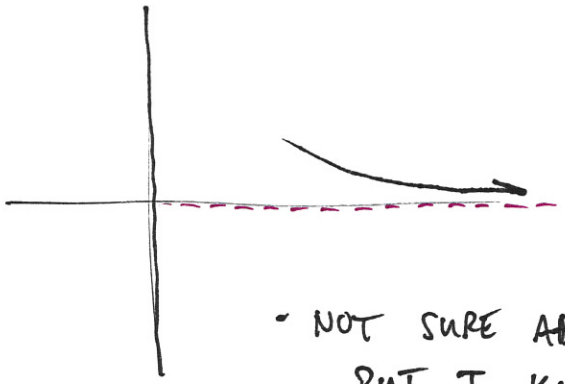
REAL PROCESS:

$$\lim_{x\to\infty} \frac{\frac{2x}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{1}{x^2}}$$

$$\lim_{x\to\infty} \frac{\frac{2}{x}+\frac{5}{x^2}}{3+\frac{1}{x^2}} \overset{DS}{=} \frac{\frac{2}{\infty}+\frac{5}{\infty^2}}{3+\frac{1}{\infty^2}}$$

$$= \frac{0+0}{3+0} = \boxed{0}$$

HEADING TO THE X-AXIS



- NOT SURE ABOUT THE REST BUT I KNOW IT TAPERS LIKE THIS....

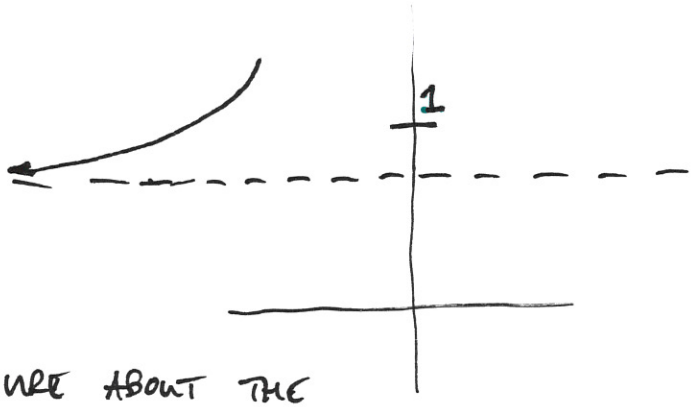
2. IF TOP DEG. = BOTTOM DEG., LIMIT OF RATIONAL IS FRACTION FORMED BY LEADING COEFFICIENTS.... HORIZONTAL LINE IS HORIZONTAL ASYMPTOTE

EX: $\lim_{x \to -\infty} \frac{2x^2+5}{3x^2+1} = \frac{-\infty}{-\infty}$ (No!!)

DS

AS YOU HEAD TO LEFT END....

L.C.'s: $\frac{2}{3}$



- NOT SURE ABOUT THE REST, BUT THIS IS WHAT LEFT END TAPERS TO....

3. IF TOP DEG > BOTTOM DEG, THERE IS NO LIMIT.... AT THE ENDS, THEY DO NOT 'TAPER' TO ANYTHING....

THAT IS DIFFERENT THAN IF ITS THE $\lim_{x \to c} f(x)$ to a # !!!!

HERE WE ARE TALKIN' END BEHAVIOR, YO...

p 78 # 1-10, (11+12), 13-18 ~~ODD~~, 25-32, 33, 34, 37-42, (43+44), 45
61-63, 75-77, 83-86, 97

p 88 # 1-4, 9-16, 19-~~26~~, 29-31, 33-40, 59, 60, 62, 63,

2 SEPARATE ASSIGNMENTS !!