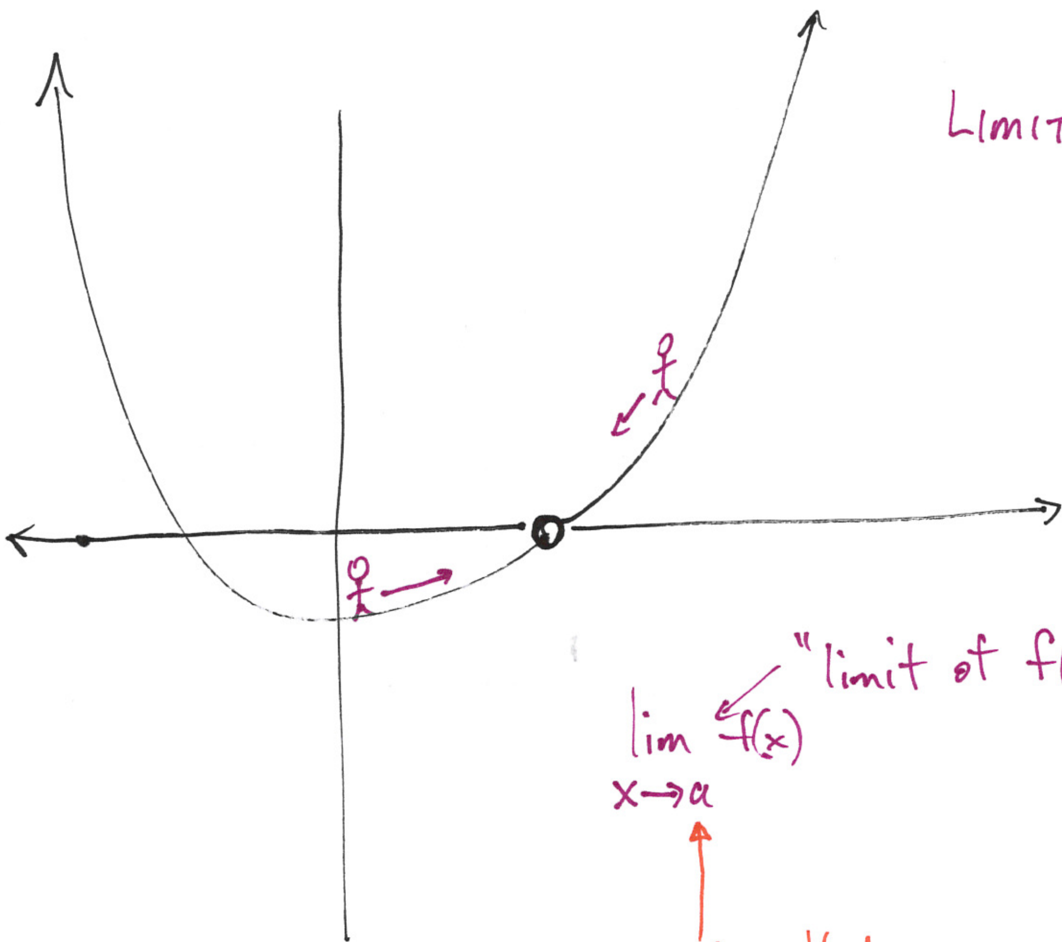


LIMIT IS Y-VALUE (HEIGHT)

"limit of f(x) as x heads to a"

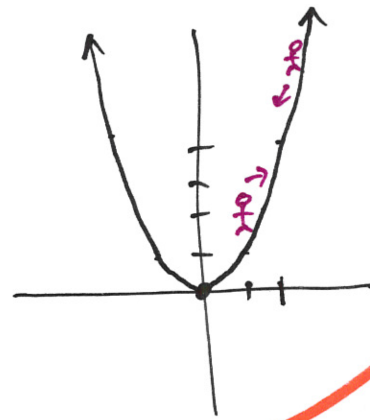
$$\lim_{x \to a} f(x)$$

x-value that we head towards $\Longrightarrow$ YIELDS WHAT Y VALUE THAT END UP AT?

EXS : 1 $f(x) = x^2$

$$\lim_{x \to 2} f(x) \Rightarrow \lim_{x \to 2} (x^2)$$

LITTLE MEN WALK EAST AND WEST, NOT UP AND DOWN



$x = 1.9,\ 1.99,\ 1.999$ EAST....

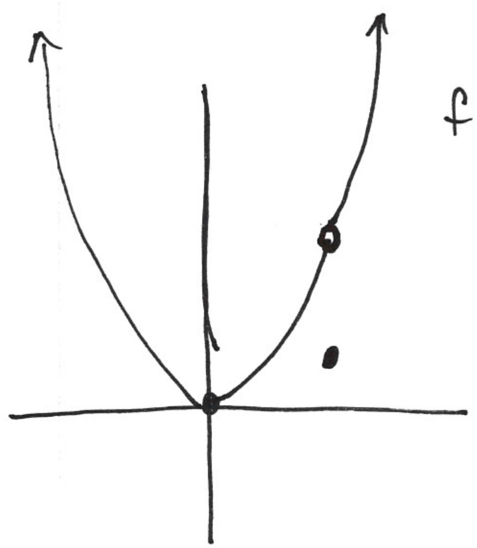
$3.61,\ 3.9601,\ 3.996001$ CLIMBING TO "4"

$x = 2.001,\ 2.01,\ 2.1$ WEST....

$4.004001,\ 4.0401,\ 4.41$

DESCENDING TO "4"

2



$$f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2 \end{cases}$$ PIECE-WISE

$\lim_{x \to 2} f(x)$ ?? STILL 4 ... DO NOT MISTAKE FOR $f(2)$ !!!

Where do the little men head?

# PROPERTIES OF LIMITS : $b, c$ are reals; $n$ is positive integer; $f, g$ continuous fncs

- CONTINUOUS → You can draw it w/o lifting your pencil....
  ↳ no holes, jumps, etc... or asymptotes...
- Continuous functions (like any polynomial fnc → ie. not rational (fraction)) are great because you can 'directly substitute' to find LIMIT
  - In these instances the value of $f(x)$ is the same as the value of the limit as $x \to a$ .... BUT THEY DON'T MEAN THE SAME THING !!

A. $\lim_{x \to c} b = b$

b is a # which means it's really a horizontal line.... There is no other 'height'.

B. $\lim_{x \to c} x = c$

Your BASIC LINE...
Plug x; y is SAME VALUE...

C. $\lim_{x \to c} x^n = c^n$

Your BASIC POLYNOMIAL....
PLUG IN X; RAISE TO POWER TO GET Y...

1. SCALAR MULTIPLES : $\lim\limits_{x \to c} b(f(x)) = b\left[\lim\limits_{x \to c} f(x)\right]$

<u>EX</u>: $\lim\limits_{x \to 1} 4x^2 = 4\left[\lim\limits_{x \to 1} x^2\right]$

2. SUM/DIFF : $\lim\limits_{x \to c} \left[f(x) \pm g(x)\right] = \lim\limits_{x \to c} f(x) \pm \lim\limits_{x \to c} g(x)$

<u>EX</u>: $\lim\limits_{x \to 3} (x^3 + x) = \lim\limits_{x \to 3} x^3 + \lim\limits_{x \to 3} x$

3. PRODUCT : $\lim\limits_{x \to c} (f(x) \cdot g(x)) = \left(\lim\limits_{x \to c} f(x)\right)\left(\lim\limits_{x \to c} g(x)\right)$

<u>EX</u>: $\lim\limits_{x \to 2} (x^2 \cdot x) = \left(\lim\limits_{x \to 2} x^2\right)\left(\lim\limits_{x \to 2} x\right)$

4. QUOTIENT : $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} g(x)}$

5. Power : $\lim\limits_{x \to c} f(x)^n = \left[\lim\limits_{x \to c} f(x)\right]^n$

<u>EX</u>: $\lim\limits_{x \to 2} x^3 = \left(\lim\limits_{x \to 2} x\right)^3$

YOU MAY NEED: LIMITS OF TRIG. FNCS

$c$ is a real,

1. $\lim_{x\to c} \sin x = \sin c$
2. $\lim_{x\to c} \cos x = \cos c$
3. $\lim_{x\to c} \tan x = \tan c$
4. $\lim_{x\to c} \cot x = \cot c$
5. $\lim_{x\to c} \sec x = \sec c$
6. $\lim_{x\to c} \csc x = \csc c$

# THE 3 (ALGEBRAIC) TECHNIQUES YOU NEED TO EVALUATE LIMITS

1. DIRECT SUBSTITUTION → ALREADY EXPLAINED → 60% of problems, but only 20% of test....

2. DIVIDING OUT (CANCELLATION TECHNIQUE): YOU BETTER BRUSH UP ON FACTORING!!

$$\lim_{x\to 4} \frac{x^2-16}{x-4}$$

** CANNOT DIRECTLY SUBSTITUTE **

Why not??

$$\lim_{x \to 4} \frac{(x+4)(x-4)}{x-4}$$ → FACTOR, IF POSSIBLE !!

$$= \lim_{x \to 4} x+4 \underset{\text{D.S.}}{=} 4+4 = \boxed{8}$$

D.S. ← good idea to put this as a clarifier....

ONCE YOU DIRECTLY SUBSTITUTE,

THEN YOU CAN DROP THE "lim",

BUT NOT BEFORE THAT !!

[P.S. YOU MUST ALWAYS BE TAKING

THE LIMIT OF 'SOMETHING'....

I better never see : $\lim_{x \to 2} =$

3. RATIONALIZATION TECHNIQUE: BUT IT'S UPSIDE DOWN... Don't fight it!

$$\lim_{x \to 1} \frac{\sqrt{3+x} - 2}{x - 1}$$

* USE THE CONJUGATE OF TOP (NUMERATOR)

$$\lim_{x \to 1} \frac{\sqrt{3+x} - 2}{x - 1} \left( \frac{\sqrt{3+x} + 2}{\sqrt{3+x} + 2} \right)$$

Now, FOIL THE TOP.... BUT USE YOUR NOODLE....

AND DO NOT ACTUALLY MULTIPLY THE BOTTOM....

TRY IT!!

$$\lim_{x \to 1} \frac{(3+x) - 4}{(x-1)(\sqrt{3+x} + 2)}$$

BY USING THE CONJUGATE, THE TOP IS A 'DIFFERENCE OF 2 SQUARES' SO "O" AND "I" ADD OUT!

$$\lim_{x\to 1} \frac{3+x-4}{(x-1)(\sqrt{3+x}+2)} \longrightarrow \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{3+x}+2)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{3+x}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

DS

'ISSUE' IS GONE!

WOULD <u>NEVER</u> SEE THAT ON CALCULATOR....

(YOU WOULD THINK ITS X-AXIS BECAUSE ITS SO CLOSE...)