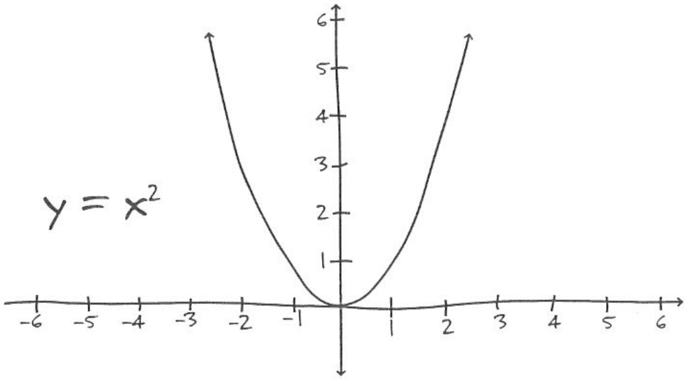
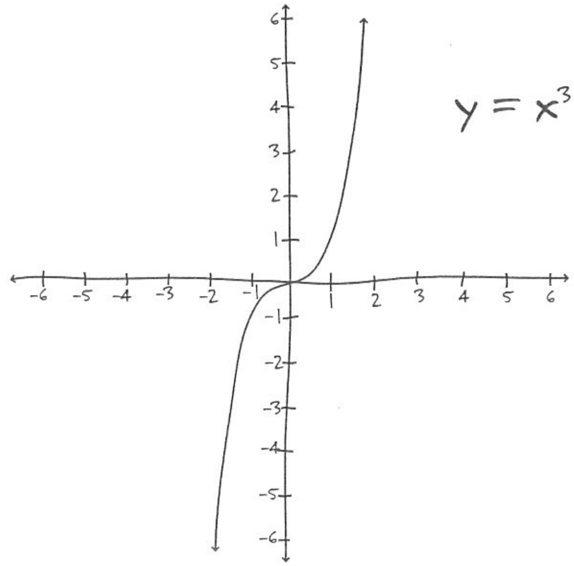
Appendix A

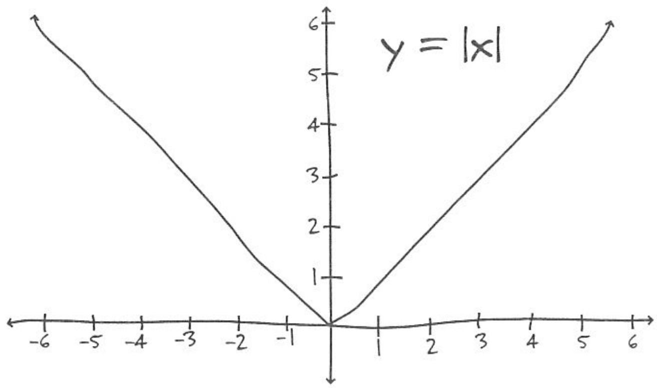
Important graphs to memorize and graph transformations



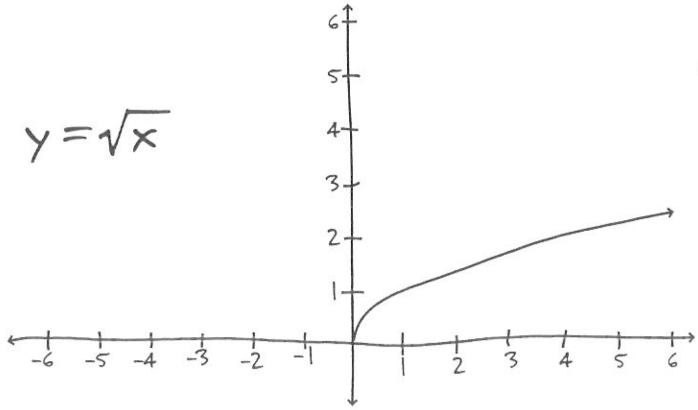
$y = x^2$



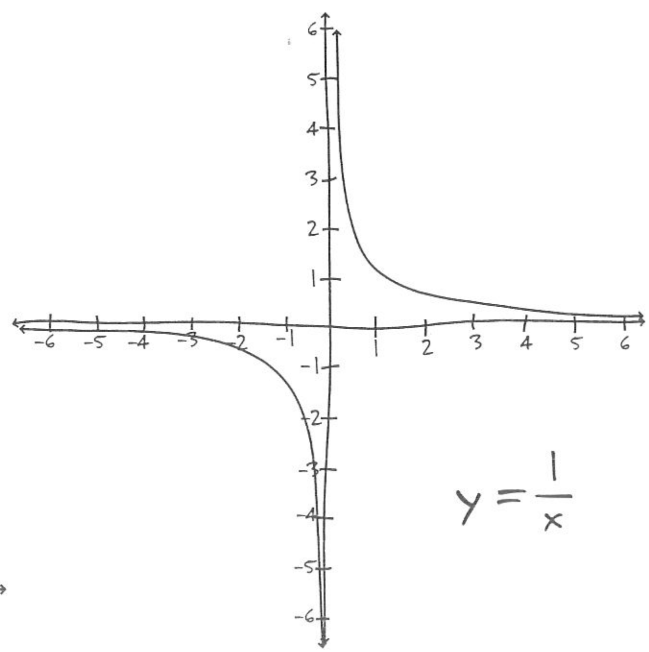
$y = x^3$



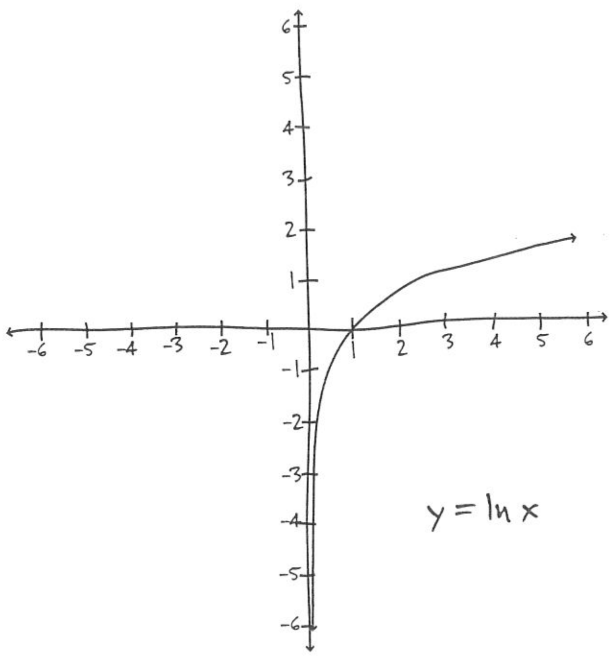
$y = |x|$

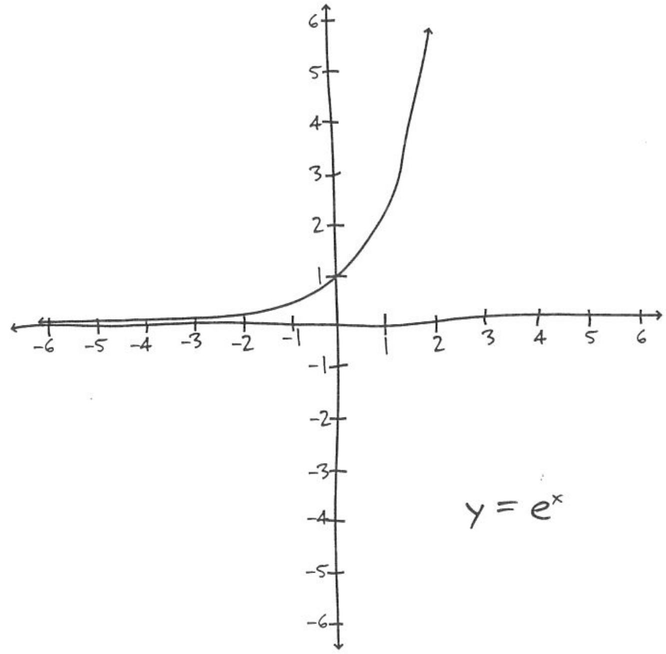


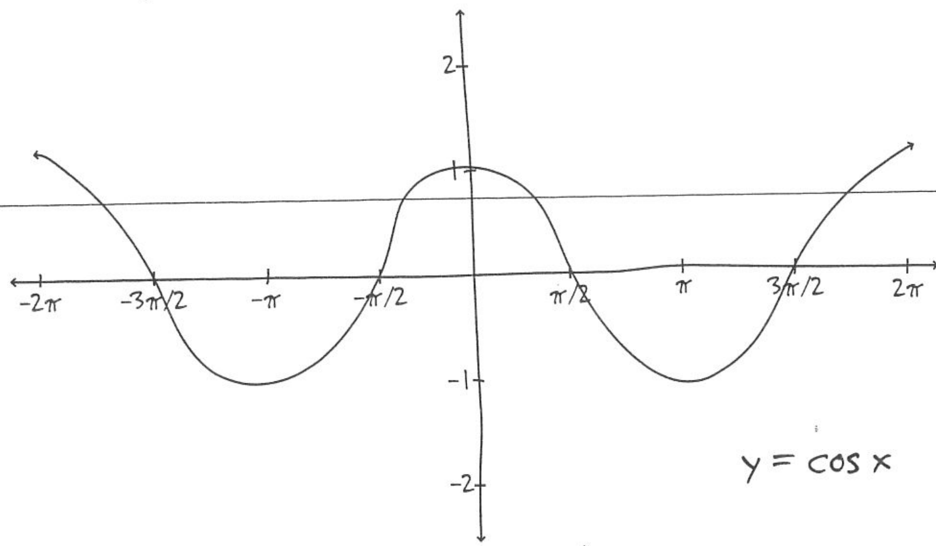
$y = \sqrt{x}$

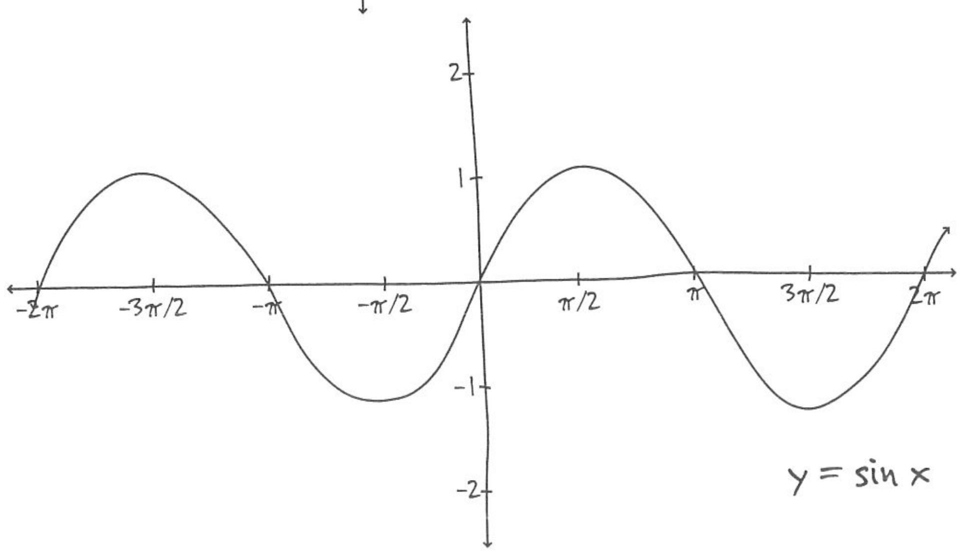


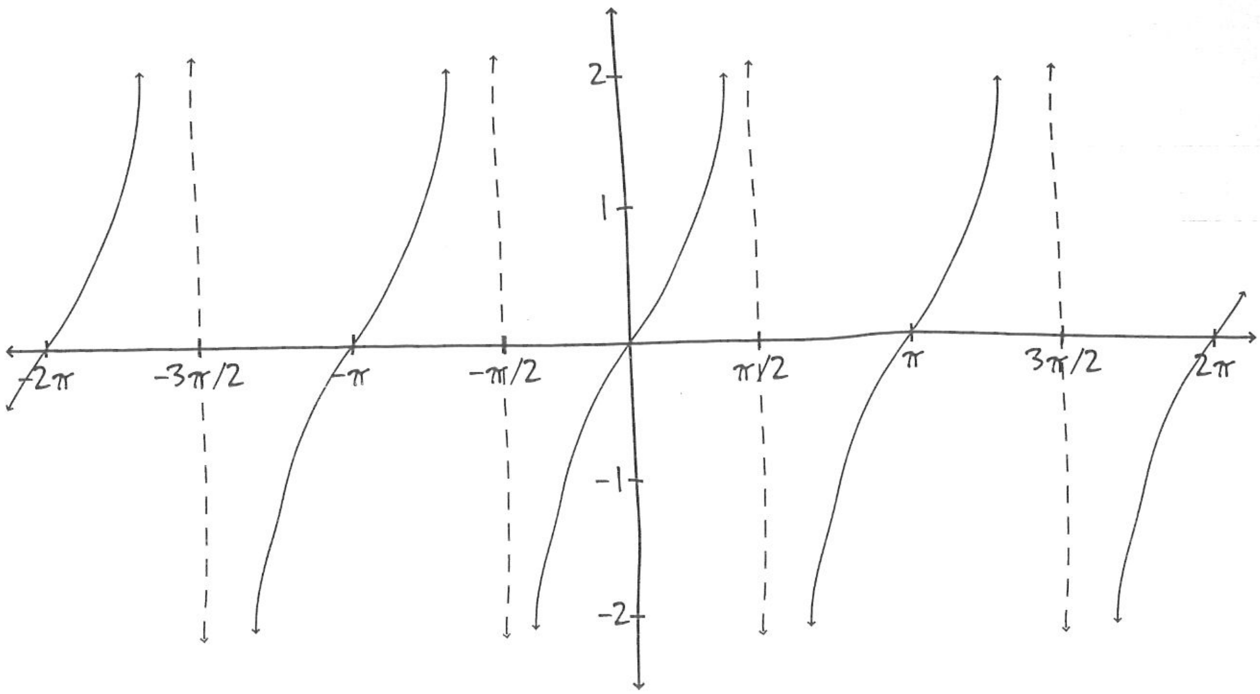
$y = \frac{1}{x}$


y = ln x
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6
6 5 4 3 2 1 -1 -2 -3 -4 -5 -6

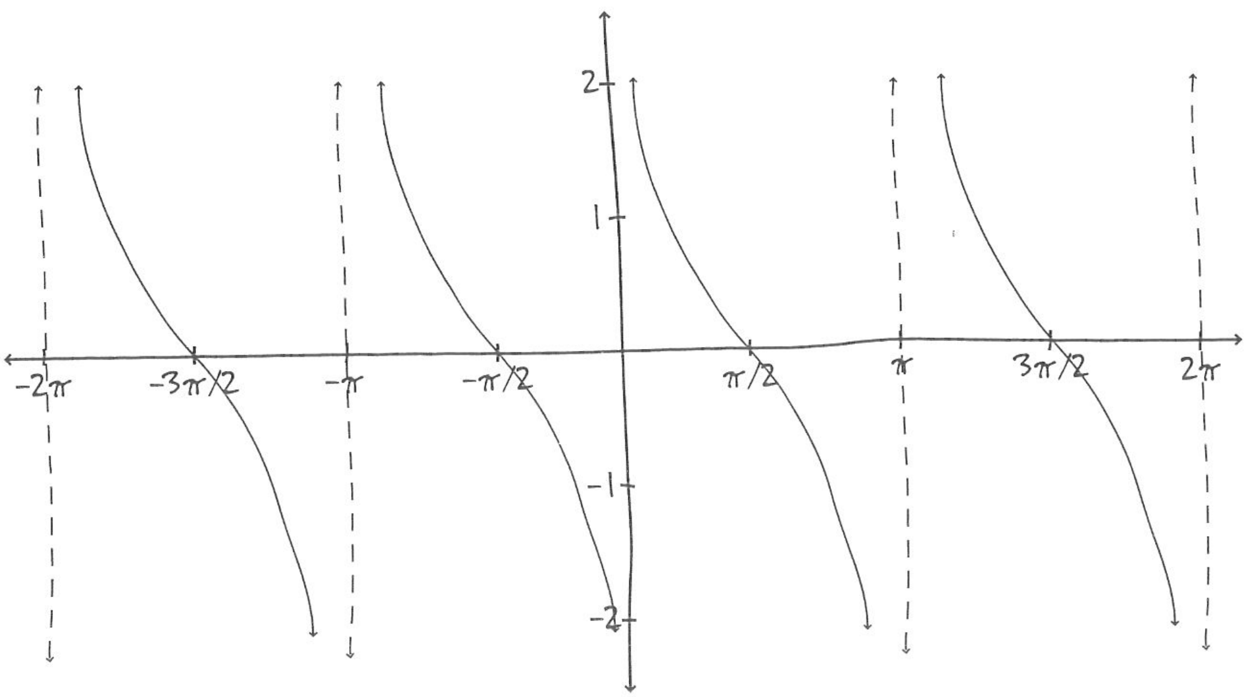

y = e^x
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6
6 5 4 3 2 1 -1 -2 -3 -4 -5 -6


y = cos x
-2π -3π/2 -π -π/2 π/2 π 3π/2 2π
2 1 -1 -2

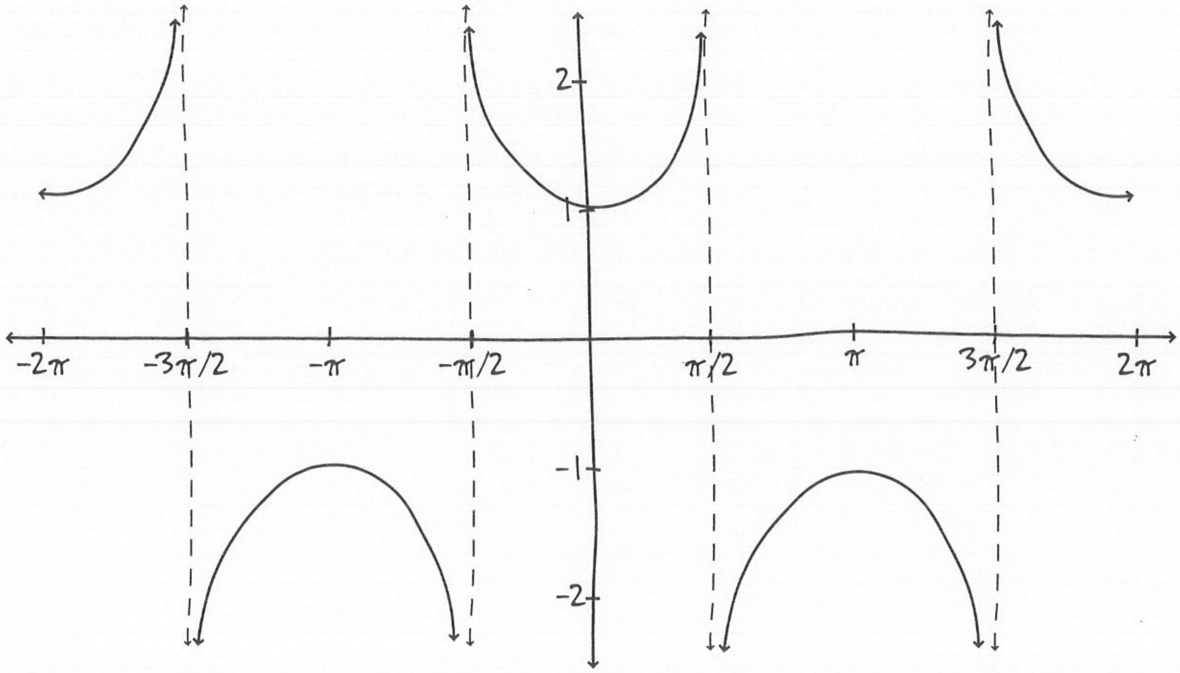

y = sin x
-2π -3π/2 -π -π/2 π/2 π 3π/2 2π
2 1 -1 -2



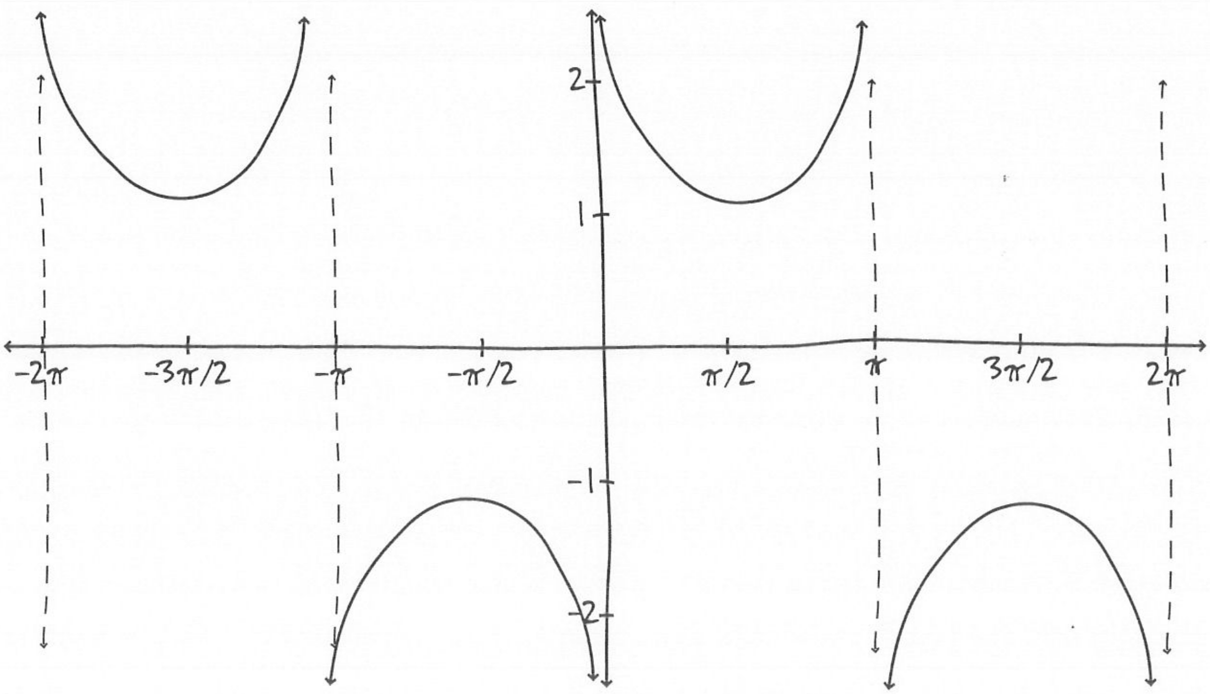
y = tan x



y = cot x



y = sec x



y = csc x

How constants transform a function graph

$$y = a \cdot f(bx + c) + d$$

Absolute value of $a \cdot f(bx + c) + d$ reflects portions of $f(x)$ below the x-axis across the x-axis

- $b > 1$: squishes $f(x)$ horizontally
- $0 < b < 1$: stretches $f(x)$ horizontally
- $-1 < b < 0$: flips $f(x)$ over y-axis and stretches it horizontally
- $b < -1$: flips $f(x)$ over y-axis and squishes it horizontally

- $c > 0$: shifts graph of $f(x)$ to the left
- $c < 0$: shifts graph of $f(x)$ to the right

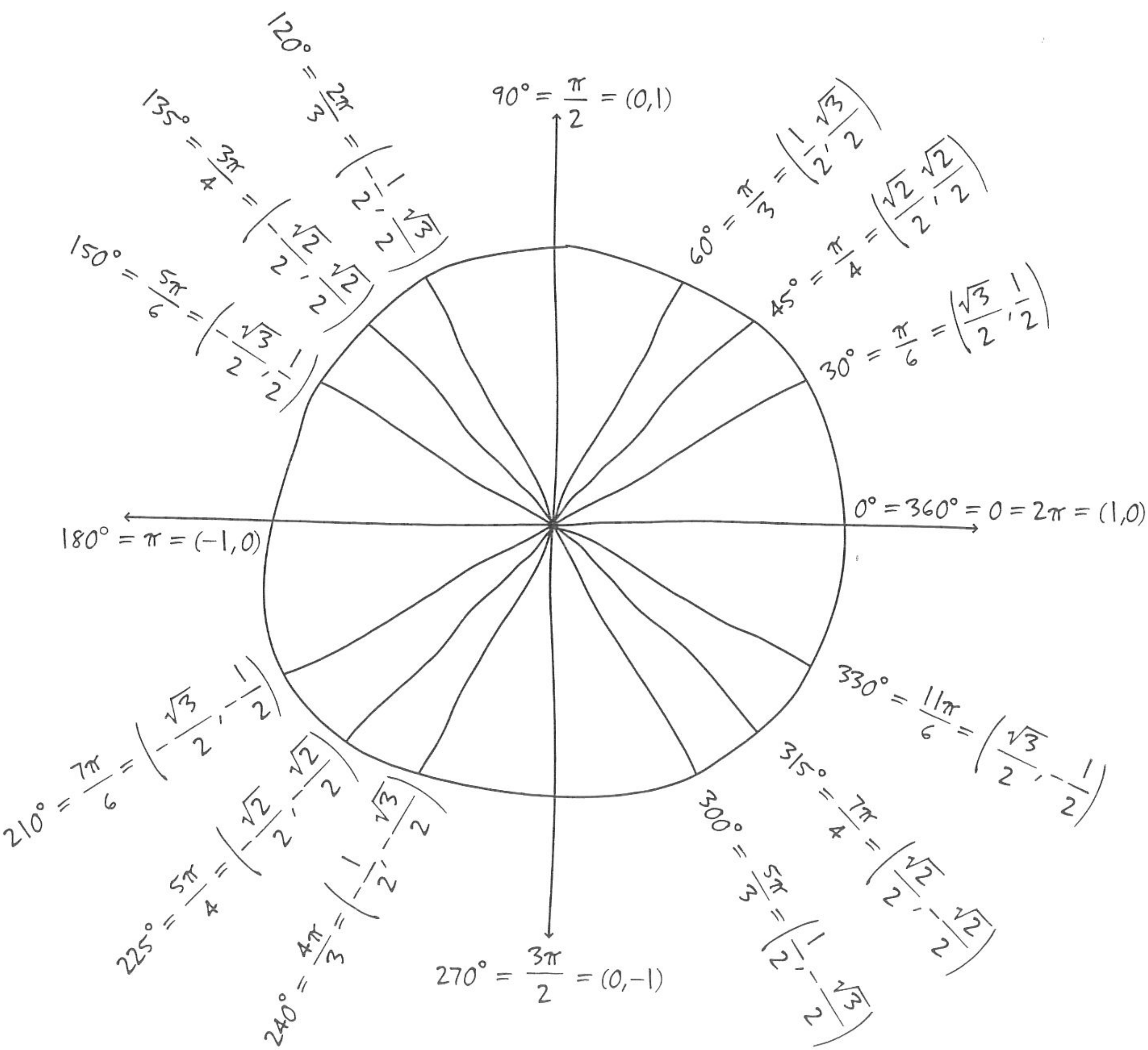
- $a > 1$: stretches $f(x)$ vertically
- $0 < a < 1$: squishes $f(x)$ vertically
- $-1 < a < 0$: flips $f(x)$ over x-axis and squishes it vertically
- $a < -1$: flips $f(x)$ over x-axis and stretches it vertically

- $d > 0$: shifts graph of $f(x)$ up
- $d < 0$: shifts graph of $f(x)$ down

Absolute value of $(bx + c)$ erases graph left of y-axis and replaces it with reflection of $f(x)$ across the y-axis

Appendix B

The unit circle



Appendix C
Trigonometric identities

Reciprocal Identities

$$\cos x = \frac{1}{\sec x} \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sin x = \frac{1}{\csc x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x} \qquad \csc x = \frac{1}{\sin x}$$

Cofunction Identities

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Pythagorean Identities

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Sum and Difference Identities

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Product-to-Sum Identities

$$\cos A \cos B = \frac{\cos(A - B) + \cos(A + B)}{2}$$

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

$$\sin A \cos B = \frac{\sin(A + B) + \sin(A - B)}{2}$$

Power-Reduction Formulas

$$\cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

Appendix D

Derivative Formulas

Trig Derivatives

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Exponential/Log Derivatives

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = (\ln a)\, a^x$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x(\ln a)}$$

Inverse Trig Derivatives

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arccot} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\operatorname{arccsc} x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

Appendix E

Antiderivative Formulas

Trig Antiderivatives

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \tan x \, dx = -\ln|\cos x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = -\ln|\csc x + \cot x| + C$$

Exponential/Log Antiderivatives

$$\int e^x \, dx = e^x + C$$

$$\int \ln x \, dx = x \ln x - x + C$$

Inverse Trig Antiderivatives

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$