Name____ Date________________________ Period______

Limits & Continuity

Short Answer: Show all work. Unless stated otherwise, no calculator permitted.

1. Explain in your own words what is meant by the equation $\lim_{x\to 2} f(x) = 4$. Is it possible for this statement to be true and yet $f(2) = 5$? Explain. What graphical manifestation would $f(x)$ have at $x = 2$? Sketch a possible graph of $f(x)$.

2. Explain what it means to say that $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 6$. What graphical manifestation would $f(x)$ have at $x = 1$? Sketch a possible graph of $f(x)$.

3. Explain the meaning of each of the following, then sketch a possible graph of a function exhibiting the indicated behavior.

   (a) $\lim_{x\to -2} f(x) = \infty$

   (b) $\lim_{x\to -3^+} g(x) = -\infty$.

4. For $f(x) = \dfrac{x^2 + x - 20}{x^2 - 16}$, algebraically determine the following:

(a) $f(4)$

(b) $\lim_{x\to 4^-} f(x)$

(c) $\lim_{x\to 4^+} f(x)$

(d) $\lim_{x\to 4} f(x)$

(e) $\lim_{x\to -4} f(x)$

(f) $\lim_{x\to 0^-} f(x)$

(g) $\lim_{x\to 1} f(x)$

(h) $\lim_{x\to -1} f(x)$

5. Using the definition of continuity, determine whether the graph of $f(x)=\frac{x^2+x}{x^3+2x^2-3x}$ is continuous at the following. Justify.

(a) $x=0$ (b) $x=1$ (c) $x=2$

6. For $f(x)=\begin{cases}-x^2, & x<0\\ 0.001, & x=0\\ \sqrt{x}, & x>0\end{cases}$, algebraically determine the following:

(a) $f(0)$ (b) $\lim_{x\to 0^-} f(x)$ (c) $\lim_{x\to 0^+} f(x)$ (d) $\lim_{x\to 0} f(x)$ (e) continuity of f at $x=0$. Justify.

7. Evaluate each of the following continuous functions at the indicated x-value:

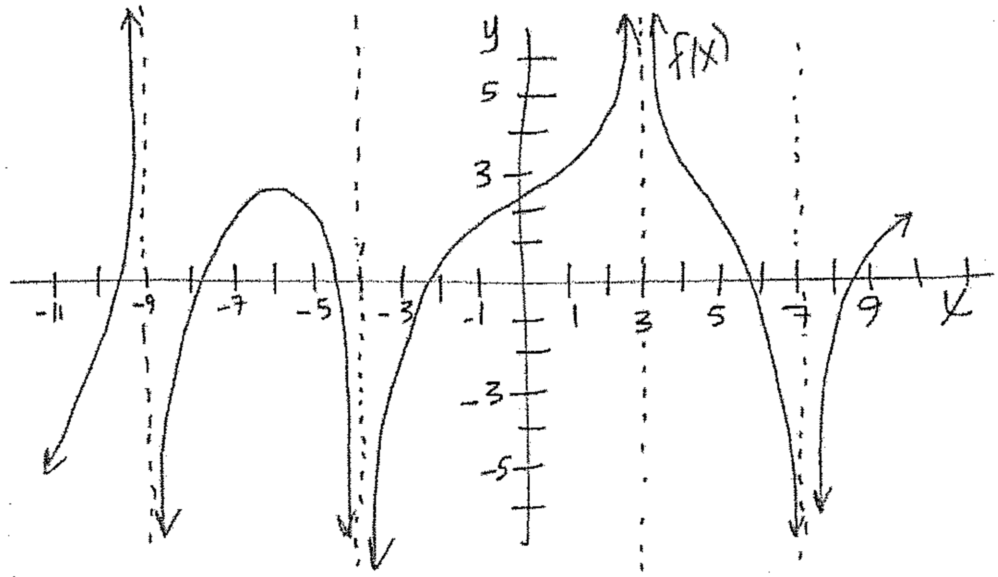
(a) $\lim_{\theta\to\frac{11\pi}{6}} \sin\theta=$ (b) $\lim_{x\to 6} 2^x=$ (c) $\lim_{x\to 0}\left(57x^{85}-2x^{45}+100x^{11}-99999x+5\right)=$

8. Evaluate each of the following:

(a) $\lim_{x\to\frac{\pi}{2}^-} \tan x=$ (b) $\lim_{x\to\frac{\pi}{2}^+} \tan x=$ (c) $\lim_{x\to\frac{\pi}{2}} \tan x=$

(d) $\lim_{x\to -5^-} \frac{-2}{x+5}=$ (e) $\lim_{x\to -5^+} \frac{-2}{x+5}=$ (f) $\lim_{x\to -5} \frac{-2}{x+5}=$

9. For the function f whose graph is given at below, evaluate the following, if it exists. If it does not exist, explain why.



(a) $\lim_{x\to 3} f(x) =$

(b) $\lim_{x\to 7} f(x) =$

(c) $\lim_{x\to -4} f(x) =$

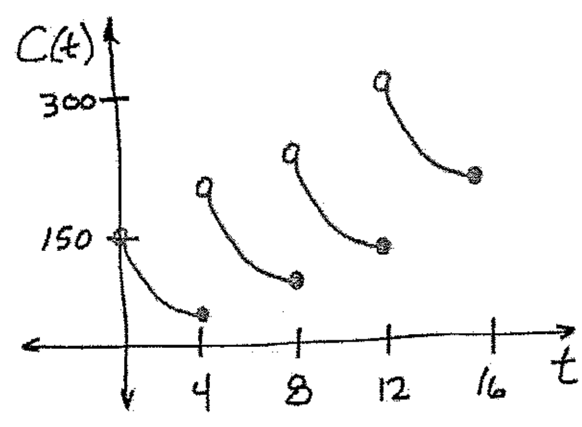
(d) $\lim_{x\to -9^-} f(x) =$

(e) $\lim_{x\to -9^+} f(x) =$

(f) $\lim_{x\to -9} f(x) =$

(g) What are the equations of the vertical asymptotes?

10. A patient receives a 150-mg injection of a drug every four hours. The graph at right shows the amount $C(t)$ of the drug in the bloodstream after t hours. Approximate $\lim_{t\to 12^-} C(t)$ and $\lim_{t\to 12^+} C(t)$, then **explain** in a complete sentence the significance/meaning of these one-sided limits in terms of the injections at $t = 12$ hours.



11. **(Calculator Permitted)** Sketch the graph of the function $f(x)=\frac{1}{1+2^{1/x}}$ in the space below, then evaluate each, if it exists. If it does not exist, explain why. Name the type and location of any discontinuity.

(a) $\lim_{x\to 0^-} f(x)=$ (b) $\lim_{x\to 0^+} f(x)=$ (c) $\lim_{x\to 0} f(x)=$ (d) $f(0)=$

12. Using the definition of continuity at a point, discuss the continuity of the following function. Justify.

$$f(x)=\begin{cases} 2-x, & x<-1 \\ x, & -1\le x<1 \\ (x-1)^2, & x\ge 1 \end{cases}$$

13. For $f(x)=\begin{cases} 3ax \quad b, & x<1 \\ 5, & x=1 \\ 2a\sqrt{x}+b, & x>1 \end{cases}$ find the values of a and b such that $f(x)$ is continuous at $x=1$. Show the work that leads to your answer.

14. **(Calculator permitted)** Fill in the table for the following function, then use the numerical evidence (to 3 decimal places) to evaluate the indicated limit. (Be sure you're in radian mode)

$$f(x) = \frac{\sin(3x)}{x}$$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$							

Based on the numeric evidence above, $\lim_{x \to 0} f(x) =$

Multiple Choice: Put the Capital Letter of the correct answer in the blank to the left of the number. Be sure to show any work/analysis.

_____ 15. $\lim_{x \to 0^-} \left(1 - \frac{1}{x}\right) =$

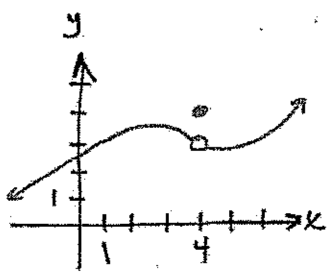
(A) 1 (B) 2 (C) $-\infty$ (D) 0 (E) ∞

_____ 16. Find $\lim_{x \to 1} f(x)$ if $f(x) = \begin{cases} 3 - x, & x \neq 1 \\ 1, & x = 1 \end{cases}$

(A) 2 (B) 1 (C) $\frac{3}{2}$ (D) 0 (E) DNE

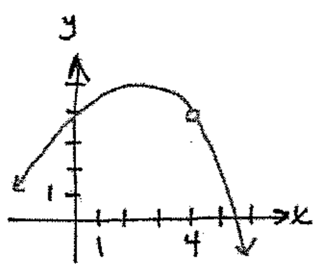
_____ 17. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I.



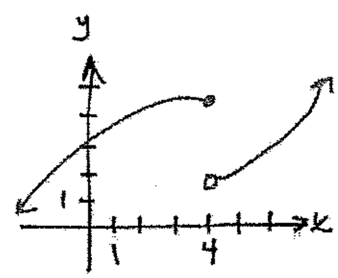
graph of $f(x)$

II.



graph of $f(x)$

III.

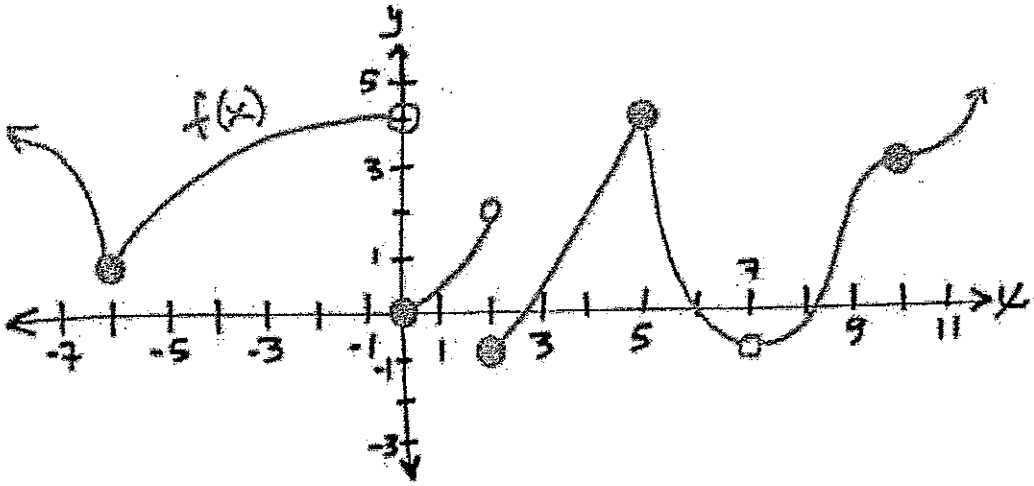


graph of $f(x)$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

_____ 18. If $f(x) = \begin{cases} \ln x, & 0 < x \leq 2 \\ x^2 \ln 2, & 2 < x \leq 4 \end{cases}$, then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



Use the graph of $f(x)$ above to answer questions 19 - 22.

_____ 19. $\lim_{x\to 7} f(x) =$

(A) 1 (B) 2 (C) -1 (D) 4 (E) DNE

_____ 20. $\lim_{x\to 0^-} f(x) =$

(A) 1 (B) 2 (C) -1 (D) 4 (E) DNE

_____ 21. $\lim_{x\to 2} f(x) =$

(A) 2 (B) 3 (C) -1 (D) 4 (E) DNE

_____ 22. Which of the following regarding $f(x)$ at $x=5$ true?

I. $\lim_{x\to 5^-} f(x) = 3$

II. $\lim_{x\to 5^+} f(x) = f(5)$

III. $f(x)$ is continuous at $x=5$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

______ 23. If $f(x)=\begin{cases} ae^{x}+b, & x<0 \\ 4, & x=0 \\ bx \quad 2a, & x>0 \end{cases}$, then the value of b that makes $f(x)$ continuous at $x=0$ is

(A) 2 (B) 2 (C) 4 (D) 6 (E) no such value exists

______ 24. If $f(x)=\dfrac{1}{x-2}$ and $\lim\limits_{x\to(-k+1)} f(x)$ does not exist, then $k=$

(A) 2 (B) 3 (C) 1 (D) -2 (E) -1

______ 25. The function $f(x)=\begin{cases} \dfrac{x^2}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$

(A) is continuous for all x

(B) has a removable point discontinuity at $x=0$

(C) has a non-removable oscillation discontinuity at $x=0$

(D) has an non-removable infinite discontinuity at $x=0$

(E) has a non-removable jump discontinuity at $x=0$

______ 26. If $f(x)=\begin{cases} \dfrac{x^2 \quad x}{2x}, & x\neq 0 \\ k, & x=0 \end{cases}$ is continuous at $x=0$, then $k=$

(A) 1 (B) $\dfrac{1}{2}$ (C) 0 (D) $\dfrac{1}{2}$ (E) 1